Methodologies of estimation of periodicities of river flow and its long range forecast. The case of transboundary Danube River

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Figure 1. Location of Orsova

Table 1. Sine approximation of runoff of Danube River, Orsova

(1840 – 1978)

T, year	Q_0, km^3/year	δQ/2, km^3/year	φ, radian	S $(km^3/year)^2$
3,0	171,7272	3,5372	4,5427	125472,1
4,0	171,7502	3,1091	2,2580	125664,2
5,0	171,7163	9,9217	,9766	119453,6
6,0	171,7223	3,8616	2,6116	125304,4
7,0	171,7422	1,2293	4,1748	126231,9
8,0	171,8295	5,3874	3,9732	124306,5
9,0	171,5987	8,5354	3,3328	121272,4
10,0	171,7359	3,1968	3,7783	125621,6
11,0	171,7688	5,3886	2,5909	124301,5
12,0	171,5867	7,1465	3,5100	122796,0
13,0	171,8779	6,9047	-,0128	123044,6
14,0	171,7533	7,8421	4,3506	122037,9
15,0	171,8974	7,9309	-,7625	121949,6
16,0	171,6274	4,3588	-1,0143	125025,5
17,0	171,7598	1,1786	2,6613	126238,9
18,0	171,6149	4,1941	3,9369	125133,2
19,0	171,5722	4,4786	1,7261	124918,4
20,0	171,7748	6,4393	2,4656	123463,7
21,0	172,0400	9,2245	-,2248	120443,4
22,0	172,0561	10,3683	,6158	118870,5
23,0	171,7898	9,3232	-,9298	120283,9
24,0	171,5374	6,6560	1,7840	123311,5
25,0	171,5883	2,7326	2,7385	125825,7
26,0	171,8046	1,5608	-,4982	126166,7
27,0	171,8769	5,3116	3,7391	124338,8
28,0	171,6753	8,5687	,6195	121269,2
29,0	171,2958	10,9611	2,7244	118280,4
30,0	170,9848	11,9497	3,8593	116746,2
31,0	170,9174	11,5970	4,0961	117050,1
32,0	171,0739	10,4950	3,5048	118531,6

Table 2. Arrangement of the minima of sine approximation of runoff of Danube River in dependence from the correlation with time series

T, year	Q_0, km^3/year	δQ/2, km^3/year	φ, radian	S $(km^3/year)^2$	r	
30,0	170,98	11,94	3,8593	116746,2	,2755	1
22,0	172,05	10,36	,6158	118870,5	,2431	2
5,0	171,71	9,92	,9766	119453,6	,2334	3
9,0	171,59	8,53	3,3328	121272,4	,2002	4
15,0	171,89	7,93	-,7625	121949,6	,1864	5
12,0	171,58	7,14	3,5100	122796,0	,1674	6

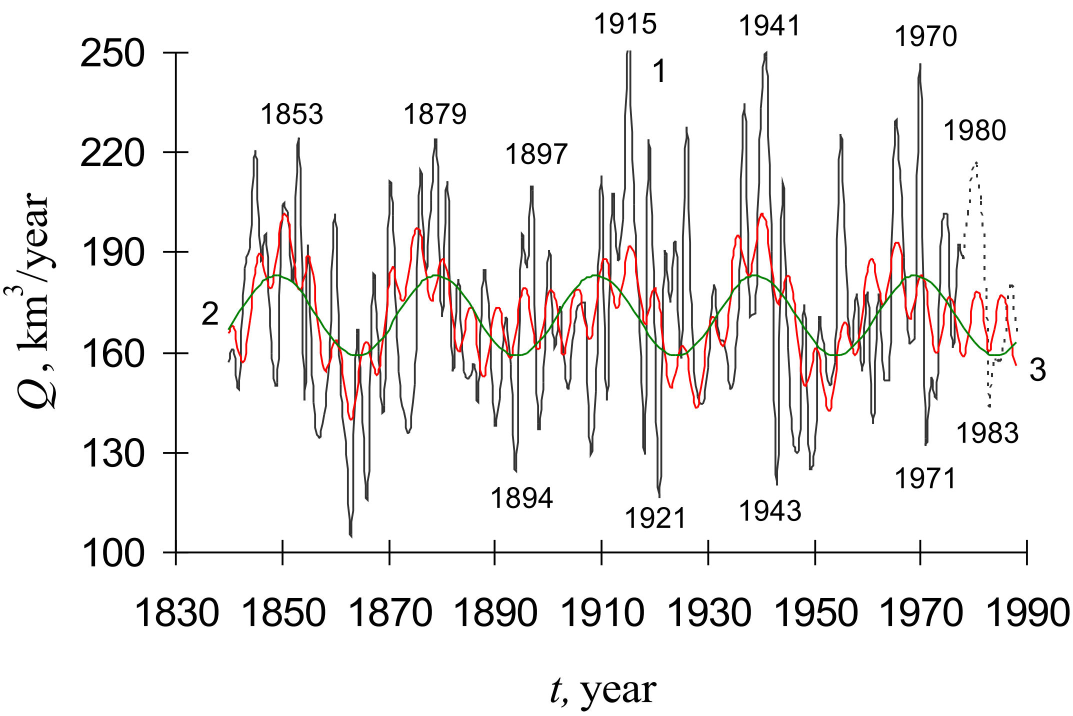

Figure 2. Variation of runoff of Dunabe River (Orsova): 1 – time series (dotted line – training forecast interval, 1979 – 1988), 2 – approximating sine with the period of 30 years, 3 – the sum of periodicities of 30, 22 and 5 years

Table 3. Statistics of runoff of Danube River, Orsova (1840 – 1978)

Mean, km^3/year	Sum of sq. diff. with mean value, $(km^3/year)^2$	Standard deviation, km^3/year	Accepted mistake, km^3/year
171,7361	126337,0	30,148	20,3197

Table 4. Estimation of results of training forecast of the runoff of Danube River

t, years	Q, km^3/year	Q_s, km^3/year	Q_s-Q, km^3/year	$(Q_s-Q)^2$, $(km^3/year)^2$	Q_m-Q, km^3/year	$(Q_m-Q)^2$, $(km^3/year)^2$
1979	201,5	166,4	-35,1	1233	-29,8	887
1980	216,9	177,9	-39,0	1523	-45,2	2046
1981	205,4	178,0	-27,4	754	-33,7	1136
1982	177,2	167,0	-10,2	104	-5,4	29,7
1983	144,0	160,3	16,3	267	27,7	770
1984	157,7	167,1	9,4	88	14,1	197
1985	157,1	177,3	20,2	407	14,6	213
1986	163,3	175,9	12,6	160	8,5	71,4
1987	180,6	163,6	-17,0	289	-8,9	79,4
1988	164,0	156,0	-8,0	64	7,7	59,3
	Mean	Mean	N. of true forecasts	Sums of squares of mistakes	N. of true forecasts	Sums of squares of mistakes
1979 – 1983	189,0	169,9	2	3882	1	4869
1984 – 1988	164,6	168,0	5	1010	5	621
Total	176,8	169,0	7	4892	6	5489

Method of Periodicities

$$R = a_0 + \frac{\delta a}{2}\sin(\omega t + \varphi) = a_0 + b\sin\omega t + c\cos\omega t .$$

(1)

$$S_a = \sum_1^n (a_i - R)^2 = \sum_1^n (a_i - a_0 - b\sin\omega t_i - c\cos\omega t_i)^2 .$$

(2)

$$\delta a = 2\sqrt{b^2 + c^2} ,$$

(3)

$$tg\varphi = \frac{b}{c} .$$

(4)

$$\frac{\partial S_a}{\partial a_0} = -2\sum_1^n (a_i - a_0 - b\sin\omega t_i - c\cos\omega t_i) = 0 ,$$

(5)

$$\frac{\partial S_a}{\partial b} = -2\sum_1^n ((a_i - a_0 - b\sin\omega t_i - c\cos\omega t_i)\sin\omega t_i) = 0 ,$$

(6)

$$\frac{\partial S_a}{\partial c} = -2\sum_1^n ((a_i - a_0 - b\sin\omega t_i - c\cos\omega t_i)\cos\omega t_i) = 0 .$$

(7)

$$\sum_{1}^{n} a_i = r, \qquad (8)$$

$$\sum_{1}^{n} \sin \omega t_i = l, \qquad (9)$$

$$\sum_{1}^{n} \cos \omega t_i = p, \qquad (10)$$

$$\sum_{1}^{n} a_i \sin \omega t_i = v, \qquad (11)$$

$$\sum_{1}^{n} \sin^2 \omega t_i = s, \qquad (12)$$

$$\sum_{1}^{n} \cos \omega t_i \sin \omega t_i = u, \qquad (13)$$

$$\sum_{1}^{n} a_i \cos \omega t_i = z, \qquad (14)$$

$$\sum_{1}^{n} \cos^2 \omega t_i = y, \qquad (15)$$

$$a_0 n + bl + cp = r, \qquad (16)$$

$$a_0 l + bs + cu = v, \qquad (17)$$

$$a_0 p + bu + cy = z. \qquad (18)$$

$$c = \frac{rsp - rul + vnu - vpl + zl^2 - zsn}{sp^2 - 2upl + u^2 n + yl^2 - ysn}, \qquad (19)$$

$$b = \frac{vp^2 - zpl + zun - upr + yrl - vny}{sp^2 - 2upl + u^2 n + yl^2 - ysn}, \qquad (20)$$

$$a_0 = \frac{r - cp - bl}{n}. \qquad (21)$$